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Touching Points on a Numeral as a Means of Early Calculation: Does this Method Inhibit Progression to Abstraction and Fact Recall?

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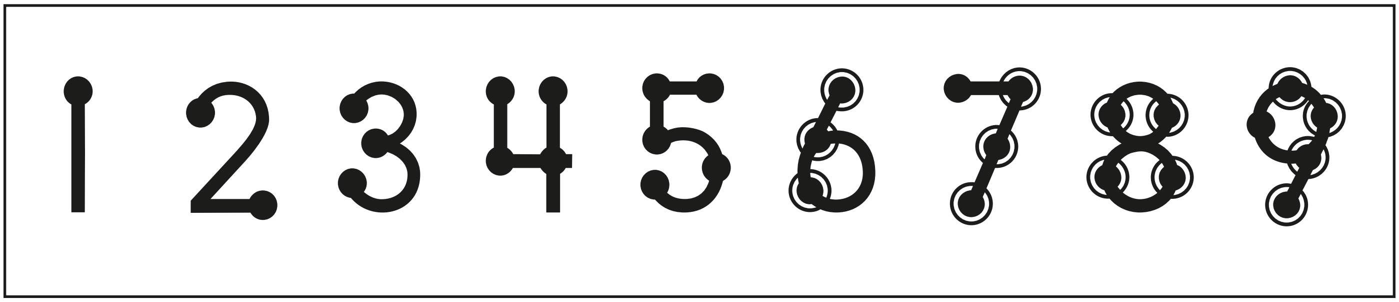
Abstract

Throughout the years, several teachers have noticed that although some students are taught TouchMath at an early age, and seem to benefit from it, they continue to count on the TouchPoints in their middle and high school years. It is important to note that these teachers are referring to a small portion of the student population. However, to these teachers, and to Innovative Learning Concepts, Inc., the developers of TouchMath, addressing these concerns is essential. Therefore, this paper will briefly describe the TouchMath process, as well as provide sample research involving students' understandings of mathematical concepts, as supported by research and developed through the TouchMath program. The TouchMath program will be demonstrated as a scaffold or instructional support that students can move away from when, and if, ready. Research involving the importance of visualizing numbers in predictable and structured ways, as well as developing and using counting procedures, will be presented as a way for teachers to modify their view of counting as a primitive, and unwanted skill. Finally, research involving 722 adult respondents, who are successful at higher mathematics, will show that counting and using strategies such as those presented by TouchMath have provided an important foundation for their success. Therefore, although it is commendable for teachers to worry about individual students' progress, that worry may be unnecessary.

Table of Contents

I. Introduction to TouchMath

Although the purpose of this paper is not to teach the reader the process of using the TouchMath method, it is important to know a few of the basics, in order to better comprehend the following sections. The TouchMath program (Innovative Learning Concepts, Inc.) utilizes TouchPoints on each numeral to represent the corresponding number (or amount). When children touch and count the TouchPoints on each numeral, they are able to tap into kinesthetic, hands-on knowledge. This action provides the young child with a sense of quantity that relates to each symbol. This also affords them multiple pathways of computing with numbers. For example, students can take numbers, such as 2 + 3, and count the total number of TouchPoints to determine the answer. Literally, students can compute as high as they can count. The TouchNumerals are shown in the box below.



Much mathematical power is in the hands of children when they learn the basics of TouchNumerals. Here is an overview of how they work. The TouchNumerals 1 through 5 have a corresponding number of TouchPoints positioned on each numeral. The TouchNumerals 6 through 9 have TouchPoints with circles around them to indicate two. These points are touched and counted twice. For example, the 6 is counted "one, two" (touching the first TouchPoint twice), "three, four" (touching the second TouchPoint twice), and "five, six" (touching the third TouchPoint twice). Other materials, for teaching with TouchPoints, are available from Innovative Learning Concepts, Inc. (www.TouchMath.com).

II. Purposes of this Study

The purpose of this paper is to investigate and address the infrequent, yet important, fears and questions from teachers about their observances of a few students in middle grades and beyond, who continue to count TouchPoints on the numerals in order to do basic computation, such as addition, subtraction, multiplication, and division. These educators do not report that these students are calculating the wrong answers. In fact, the students are producing getting the correct answers. The teachers simply are not in approval of the method the students are using. The fear is that these children are "too old" to be "counting the answers," and should be taught to "move away from" touching points and calculating. In an attempt to move students beyond this, some teachers require students to repeatedly write their number facts, engage in flashcard instruction, and participate in "games" that require students to know the answers to number facts in order to win or to continue playing. Much of this effort to push students toward the abstract level of thinking, results in frustration among teachers and students.

In analyzing the concerns of teachers, a flood of questions arise. Is it possible that the TouchMath method inhibits students from progressing to the abstract level? Do the means of teaching and using TouchMath justify the end results? Does the TouchMath method cause dependency on counting and calculating answers (as opposed to mental math and fact recall)? Maybe the most important questions are the following: What strategies would these students employ if they didn't have TouchPoints? Would these same students simply "shut down" when asked flashcard facts, if they didn't have TouchMath? Are these students in the upper, middle, or lower third of their class, with regard to mathematical abilities and achievement? This paper provides answers to teachers' concerns by an investigation of the research literature, as well as a demonstration of a link between foundational research and the soundness of using

TouchMath with students.

III. Sample Research Involving Number Facts and Mathematical Calculations

It appears overly sensitive to focus on students' inabilities to provide quick answers to questions such as, "What is 3 times 4?" Studies have shown that too often many teachers focus on fast answers to simple questions. In fact, the best evidence points to using a balance of low- and high-level questions, and wait times of three seconds after asking questions of a high level (Askew & Wiliam, 1995; Redfield & Rousseau, 1981; Tobin, 1986). For example, "What is 3 times 4?" is a low-level question. An example of a high-level question would be, "How would you draw dots to show 3 times 4?" Mental calculation and rapid fact recall are not the most important mathematical abilities. In his opening statement, Sugarman (1997) stated, "Children who are able to calculate mentally are often found, when asked, to have adopted strategies that they were not taught." He further states that many teachers have decided to focus less on paper and pencil procedures, and more upon helping students develop a "feeling for a number."

It is important for students to learn to use sources other than memory, symbols, and numerals in mathematics. Memory is not always reliable, and numerals and symbols are at the high end of representing mathematics. According to Bruner (1963, 1966), we develop mentally in three stages: concrete (using hands-on experiences), pictorial (using pictures), and symbolic (using letters and numerals). It is important to note at the beginning, that although ages are linked with three stages, learners at any age will benefit from experiences beginning at the concrete level when learning a new concept. According to Piaget (1975), children develop mentally in four stages. In the sensorimotor stage, the only way for children to acquire knowledge is through their own concrete actions and experiences. In the preoperational stage, children can use words to learn. For example, we might say that the drink is "hot." The child doesn't have to touch the coffee cup to know what the concept of "hot" is. In the concrete operations stage, children can learn abstract or symbolic concepts (such as $2 + 3 = 5$), as long as they relate them to manipulatives. In the formal operations stage, mental representations of mathematical concepts and processes, apart from manipulatives, are possible.

TouchMath employs the research of Bruner and Piaget by taking the numerals (Bruner's stage 3) and placing TouchPoints on them (Bruner's stages 1 and 2). Therefore, when students are working with numerals, they still have the visual representation of quantity, as well as counters, if necessary. The last two words of that sentence are probably the most important - "if necessary." Students who are still counting TouchPoints, beyond the time that teachers think they should, are possibly those who have not progressed beyond Piaget's concrete operations stage, no matter what their ages. In fact, much research shows that there are adults who function fairly well mathematically, yet remain in need of counters. Therefore, we shouldn't be surprised to find middle- and high-school students still dependent upon using TouchPoints.

IV. Scaffolding Students' Mathematical Computation

TouchPoints on numerals serve as scaffolds or supports for student learning. Vygotsky (1962, 1967) is known for his theory of scaffolding. In construction, scaffolds allow workers to attain heights that they would not be able to otherwise reach. As they complete their construction, they require fewer and fewer supports. This is the same case with Vygotsky's concept of instructional scaffolding (Vinson, 2001). Bridging this with the above-mentioned research of Bruner and Piaget, children will need manipulative activities, as well as pictures, to allow them to work on the symbolic level. These objects and pictures, or TouchPoints on numerals, serve as scaffolds. It cannot be expected of young learners to effectively and accurately work with symbols in the absence of instructional supports.

Sugarman's investigations with students have shown that children develop a variety of strategies that their teachers did not teach them, and that they employ these strategies when their memory fails to carry them through the problem (1997). Therefore, even when teachers insist that students follow a particular method, students may not conform. Similarly, when teachers insist that students stop doing such things as using their fingers, students will continue to rely on their most comfortable mode of computation. It is the teacher's responsibility to scaffold instruction so that students are provided with patient support up to the "height" that they are able to reach, realizing that not all students will develop the ability to memorize all basic facts, with rapid recall. Some children will need to rely on TouchPoints, finger counting, picture drawing, and a variety of other self-determined methods.

Koshy (1994) studied children's ways of doing subtraction. When reflecting on one student's method, Koshy wondered if the child rejected the algorithm proposed by the teacher and the textbook, in favor of his own way. Another child demonstrated an invented algorithm for subtraction that provided ten dots in the ones column, when ten was regrouped or borrowed. A child who worked the problem incorrectly evidenced much confusion when asked to explain that method. The researcher surmised that the child had not linked the manipulative experiences with the algorithm taught. Therefore, he could not rely on his knowledge of the manipulatives when doing the algorithm without the manipulatives. Another child had mixed methods he was taught by three people into one incorrect method. Since he didn't understand the methods he was taught, a mixture of the three was just as good as mimicking any one of them. Koshy's research demonstrates, in part, that children need consistent methods of instruction. TouchMath's consistent verbal steps for working problems, as well as using the TouchPoints on the numerals, provide a much-needed scaffold for struggling students.

V. The Importance of Visualization and Counting

Kline (1998), a teacher of kindergarten for many years, used various manipulative activities, but found that kindergarten mathematics should be more than counting. Kline's problem was that after a wealth of manipulative activities, some children did not develop fluency with numbers; instead, they focused on counting concepts. Her fear was that the manipulatives actually caused them to focus on counting by ones. In order to facilitate students' move from counting strategies to mental imagery with numbers, Kline read articles on the subject by Baroody and Standifer (1993), Payne and Huinker (1993), Van de Walle (1990), and Wirtz (1980). In each of these, the authors suggested using patterns of numbers in a visual format, such as those found on dice, dominoes, and ten-frames. She began to use "quick image" activities at the beginning of each class session. She flashed an arrangement of objects or pictures and then removed it from sight. She asked children to draw pictures of what they saw, to describe them orally, to hold up a numeral card that represented the number they saw, or to hold up the number of fingers that represented the amount they saw in the quick image. She noted that quick image activities should not replace manipulative activities, since concrete materials help children gain fluency with numbers. However, she did add that the quick image activities helped children to move beyond their one-to-one counting of sets to thinking about numbers as combinations of various smaller sets. This allowed students to relate counting, and number flexibility, to the addition process as they discovered various number combinations. Therefore, the visualization of the TouchNumerals may be necessary for some children to move beyond counting of the actual TouchPoints. If they see the TouchPoints in their minds, then they may not need to have them on paper; thus providing for greater abilities to work accurately with number facts.

Much of the computation with TouchNumerals involves counting TouchPoints. Counting is a very important foundational activity for all mathematics - such as addition and subtraction. The result of adding two counting numbers is another counting number. The result of subtracting a smaller number from a larger number is also a counting number. The studies by Groen and Resnick (1977) showed that children as early as preschool are able to invent ways of calculating, many of which are counting strategies. Carpenter and Moser (1983) conducted a long-term study with children from six to nine years old, and found that they pass through a series of five stages when determining the sum of two numbers. Each of these stages is incorporated in the series of activities set forth in the TouchMath program.

(1) counting all - children count 3 objects and 4 objects together to determine 3 + 4;
(2) counting on from the first number - children say the first number "three" and count forward the number of times for the second number "four, five, six, seven";
(3) counting on from the larger number - children count forward from the largest number, instead of the first number, "four" and then "five, six, seven";
(4) using known number facts - children provide a correct response from memory, rather than by counting or other means; and,
(5) using derived number facts - children use a number fact they know from memory and then calculate another number fact based on that, such as 2 + 2 is 4, therefore, 2 + 3 is one more than that, which is 5.

Houlihan and Ginsburg (1981) showed that children often use counting strategies to correctly answer simple addition and subtraction problems, and subsequently use their self-taught counting strategies to solve problems. Thompson (1997) studied research reports and summarized that counting involves all the following subskills: (a) reciting of the standard counting word sequence up to a certain number and in a correct order, (b) matching number names in one-to-one correspondence with the objects or pictures to be counted, (c) ensuring that the same number name is not matched to two different objects or pictures, (d) ensuring that each object to be counted has one, and only one, counting name assigned to it, (e) matching the recitation of the number names with the action of moving or pointing to the objects or pictures (46). In the process of counting, Thompson further noted that children are developing a sense of number, as well as cardinality. Thompson wrote that, "there is not doubt about the extensive use that pre-school and early years children make of counting skills when solving problems involving mental arithmetic calculations, particularly in situations that make 'human sense' to them" (48). Thompson further noted that in his extensive studies, he has found that children rely heavily on counting as a part of their "problem solving armory," furthermore, combining counting with other skills to solve increasingly complex problems. Thus, the importance of counting cannot be overlooked. Counting is not the "cheating" way of finding an answer; instead, it is the most commonly used method, providing the greatest number sense and accuracy.

V. Successful Adults' History of UsingCounting Points for Computation

Upon first hearing the concerns from a few teachers that children in middle and upper grades continue to use TouchPoints, I began to query my undergraduate early childhood and elementary mathematics methods students, as well as participants in the inservice workshops I conducted. After briefly introducing participants to TouchMath, as described in the first section of this paper, I asked, "How many of you use or have used TouchMath as a student or as a teacher?" After the first question was asked and answered, one or more participants would invariably tell about their experiences with invented strategies they employed, which were, to their surprise, strikingly akin to TouchMath. Therefore, I always followed with a second question, "How many of you used a method similar to TouchMath?" The short explanations that followed the responses to this question could be summarized as using tallies, dashes, and various other symbols drawn on or nearby the numeral. Most reported that they had designed dots on numerals in a similar fashion to those used with TouchMath.

The results of the answers to the two questions are shown in the table below. Note that the information is summarized from six semesters and four inservice workshops, involving a total of 722 respondents.

Table of Results		
Total respondents 722		
1. Use or used TouchMath	"How many of you use or have used TouchMath as a student or as a teacher?"	274 or 36%
2. Used a similar strategy	"How many of you used a method similar to TouchMath?"	231 or 32%
3. Used TouchMath or a similar strategy as a student	(Combination of respondents to questions 1 and 2)	505 or 68%

It could be surmised that the respondents are all functionally successful in mathematics, since the undergraduate students had all passed a minimum of three college-level mathematics courses, and the inservice participants had college degrees with at least one year of teaching experience. Since the location of the university is close to a city serving the space industry, many students follow their parents to the area and represent a well-educated background, sometimes from overseas, while other students from the space industry choose education as a second career. Notice from the table that the majority (68%) of respondents either used TouchMath or devised a similar strategy. Therefore, this research reveals that many successful adults have either used TouchMath or some self-initiated variation of it.

VII. Using Structured Representationsfor Number Concepts

Part of the effectiveness of TouchMath is the use of a consistent, structured pattern of TouchPoints on the numerals. Chao, Stigler, & Woodward (2000) compared the use of specific patterns to represent numbers (such as ten-frames), as opposed to using a variety of patterns with diverse objects for representing numbers. Although both methods were found to be effective in their own ways, it is important to note that the structured materials, like stationary TouchPoints on numerals, facilitated children using strategies for computation other than their fingers, as well as sped up their answers. Furthermore, they found that using structured representations of numbers helped children develop more sophisticated strategies, such as nonfinger computation strategies. Overall, research results suggested that structured materials seem to be a better tool than using a variety of materials. The researchers concluded that children's mental images were more consistent when they internalized one structured representation of the numbers, as opposed to using a variety of interpretations, especially when presented with multiple exposures. Although this study dealt with addition, the researchers noted that similar results have been found with subtraction, which is more advanced than addition (Jordan, Huttenlocher, & Levine, 1992; Levine, Jordan, & Huttenlocher, 1992).

Chao and colleagues (2000) explained that there are two basic views of numerical representations: the mental tool view and the abstraction view. The mental tool view proposes that children develop mental images of numbers from repeated exposure and use of physical or pictorial materials that represent those numbers (Hiebert, Wearne, & Grant, 1994; Stigler, 1984). Children develop the mental image of the number from these experiences, and it forms a basis for their understanding of numbers as they work with more complicated numerical problem solving processes. Some educators and researchers believe that using one well-selected representation of number concepts is beneficial (Baroody, 1989; Flexer, 1986; Hatano, 1982; Howden, 1989; Nesher, 1989; Stern, 1949; Thompson & Van de Walle, 1984; Wirtz, 1978).

The mental tool view relates most closely with TouchMath, since TouchPoints are used on numerals in a consistent way, so that the visualization of these numerals serves as a “tool” for them. When students don’t have counters available, they still have the “counters” (or TouchPoints) on the numerals.

Dienes (1964) proposed the abstraction view, which places emphasis on the student gaining mathematical understanding from the concrete representations. The research of Van de Walle (1988) and Baratta-Lorton (1972) with her “Mathematics Their Way” program, have further proposed that students need a variety of perceptual experiences in order to form the soundest mathematical understanding. The study by Chao and colleagues suggested that the mental tool view was more consistent with their findings, since repeated exposure to one representation of numbers helped children perform better in addition, than those who used multiple representations.

The whole of this research tends to support TouchMath’s use of TouchPoints for computation. Children who used structured images had better recognition times and seemed to have mentally encoded the structured images. In contrast, the children who used various representations of numbers seemed to not have any particular encoded image of a number. In effect, students would be better able to remember structures for numbers, such as TouchPoints on a numeral, as opposed to using dominoes, counters, Dienes Blocks, and a variety of other representations.

VIII. Conclusions

The main purpose of this paper was to address some teachers’ concerns about the few middle and high school students who continue to derive answers to basic computation through counting of TouchPoints. Taking the literature reviewed, whether in part or as a whole, counting and using a consistent representation of a number seems to provide the best results. TouchMath is based on the soundest research, from the foundational writers, such as Piaget and Bruner, through the most current research. As was shown in the fifth section of this paper, even successful adults have a history of either using TouchMath or some self-initiated variation of it. In fact, the TouchPoints seem to have some intuitive value in them, as is often pointed out by people who see the TouchNumerals for the first time, recognizing the closeness of TouchNumerals to their own invented methods. These people provide consistent support that working on the concrete level does not inhibit the development of mental strategies, or inhibit them from moving to the symbolic level. Just as some people need eyeglasses to see, a few students may continue to count the TouchPoints in order to “see” the mathematics. The few older students, who need to use TouchPoints as scaffolds, should not diminish the credibility of the method itself. In fact, it should lend to its credibility, since these students may not have any support system without the use of TouchPoints.

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